QP CODE: 26022399

26022399

Reg No : .....

Name : .....

M Sc DEGREE (CSS) EXAMINATION, MAY 2026

Second Semester

M Sc MATHEMATICS

CORE COURSE - ME010204 - COMPLEX ANALYSIS

2019 ADMISSION ONWARDS

FCEC078B

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. What is Hadamard's formula for radius of convergence? If $\Sigma a_n z^n$ has radius of convergence R, what is the radius of convergence of $\Sigma a_n z^{2n}$?
2. Define arcs and closed curves in a complex plane and give examples.
3. Evaluate $\int_{|z|=2} \frac{dz}{z^2-1}$ for the positive sense of the circle.
4. What do you mean by rectifiable arcs?
5. Prove that the index n(γ,a) is zero if 'a' lies in the unbounded region determined by the closed curve γ.
6. State the Cauchy's integral formula for higher derivatives. Evaluate $\int\limits_{|z|=2} \frac{z^2}{(z+1)^3} dz$.
7. Prove that $z = -1$ is an essential singularity of the function $f(z) = cos\frac{1}{z+1}$.
8. State the local mapping theorem. Use it to prove that $\int_\gamma \frac{z+1}{z^2+2z-8} dz = 0$ where γ is the unit circle.
9. Define a simply connected region.
10. State Rouche's theorem.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Calculate the distance $d(z.z')$ between the stereographic projections of z and z' in the complex plane.
12. Prove that the set of all linear transformations form a group under composition of mapping.
13. Evaluate $\int_{|z|=1} \frac{z}{z^2-4} dz$.
14. Evaluate $\int_{|z|=R} \frac{|dz|}{|z-a|^2}$, where $|a| \neq R$
15. Let $f(z)$ be meromorphic in a region Ω and $z=a$ be a pole of order n. Prove that there exists an analytic function $g(z)$ in Ω such that $f(z)=(z-a)^{-n}g(z)$, where $g(a) \neq 0$.
16. State and prove the maximum principle.
17. Explain the method for calculation of residues.
18. Evaluate $\int_0^{\pi} \frac{d\theta}{2+\cos\theta}$.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. (i) Reflect the imaginary axis, the line $x=y$ and the circle $|z|=1$ in the circle $|z-2|=1$.
    (ii) State and prove the symmetry principle for linear transformation.
20. State and prove Cauchy's theorem for a rectangle.
21. (a) State and prove the Taylor's theorem.
    (b) Derive the integral expression for $f_n(z)$ in the remainder term.
22. If Ω is a region with connectivity n, then construct a homology basis for all cycles in Ω and discuss about periods of the indefinite integral.

(2×5=10 weightage)